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ON THE MECHANISMS OF ENERGY AND MOMENTUM TRANSFER

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A relaxation model describing energy and momentum transfer in a substance is proposed and its physicomathematical substantiation is given. Based on the model, universal temperature relations for thermal conductivity and viscosity are proposed. An attempt is made to extend the relaxation model to phenomena of turbulent transfer in gases and liquids.

A great number of models have been developed to describe the mechanisms of energy transfer by heat conduction and momentum transfer through dynamic viscosity as applied to gases, liquids, or solids. Various quantitative relations on their basis have been proposed. However, their analysis shows that all of them are of a semiempirical character, since each such relation is applicable only in a small temperature range for a narrow group of substances and only for a single aggregate state. Therefore, all these relations contain numerical empirical coefficients obtained by comparing the proposed model with experimental data. But some attempts have been made to substantiate these empirical coefficients theoretically. This is, for instance, the case with the Lorentz coefficient in the Wiedemann–Franz law, which relates the thermal conductivity of conducting substances to their electric conductivity. It is well known that this law is absolutely inapplicable at low and high temperatures, even where different Lorentz numbers are used. In our opinion, this is explained by the radically incorrect assumption that the total heat is transferred in conductors only by conduction electrons. It is quite obvious that heat as a kind of energy (differing from electrical energy) supplied to a body is received and transferred by all microparticles constituting the given substance and primarily by atoms and molecules. Some efforts known in the literature to take into account this circumstance by applying the additivity rule to different particles (molecules, atoms electrons, photons, photons, excitons, etc.) have not given and could not give an accurate account of transfer processes, since in this case the possibilities of their mutual scattering are excluded.

Our model implies that in processes of energy and momentum transfer directly (i.e., direct transfer of energy or momentum) or indirectly (i.e., energy or momentum is transferred by scattering particles) all particles and defects of a body in its any aggregate state participate. According to the current physical concenpts of transfer and scattering processes we consider that the maximum possible transfer rate is equal to the sound velocity (V_s). We consider that the basic quantity which determines the scattering processes in any aggregate state is the relaxation time, which is defined as the period within which energy or momentum is freely transferred between two particles or between a particle and a defect. To determine the average minimum relaxation time, we propose to calculate it in any aggregate state of a substance with allowance for the temperature distribution using the following relation [1]:

$$\bar{\tau}_0 = \frac{\hbar/(kT)}{\exp(T^*/T) - 1}, \tag{1}$$

where $\hbar = h/2\pi$ ($\hbar$ is the Planck constant); k is the Boltzmann constant; T is the characteristic temperature. We assume that the characteristic temperature for solids is equal to the temperature at maximum thermal conductivity. It is obvious that these values depend, first of all, on the structure perfection of a solid: the maximum thermal conductivity of perfect single crystals is observed, according to the literature data, at ~2 K, and that of amorphous

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bodies, at the melting temperature. For liquid and gaseous bodies we assume that the characteristic temperature is equal to the temperature of the "liquid–gas" phase transition.

However, most often the real mean relaxation time in solid, liquid, and gaseous substances is greater than the minimum values determined by relation (1). Here, the matter is that not every each scattering event changes the energy or momentum in a transfer process. Thus, it is well known, for instance, that two-phonon scattering processes do not change either the frequency or wave vector, i.e., are not capable of exerting an influence on the amount of energy and momentum transferred, and three-phonon or "normal" processes of transfer change momentum but do not influence the amount of energy transferred. These examples can be continued for scattering by defects, scattering of the "phonon–electron" type, etc., since a change in the transferred energy or momentum is possible only for a definite ratio of frequencies and wave vectors.

Therefore, we consider that the processes of energy and momentum transfer are determined by the mean real relaxation time $\bar{\tau}_r$, which is defined as the time between two events of scattering when either energy or momentum changes. We propose to determine $\bar{\tau}_r$ on the basis of its probability representation relative to the maximum possible relaxation time τ_{max} for real bodies of finite dimensions in any aggregate state which have been assumed to equal 1 sec. Then, using the representation of statistical unexpectedness and assuming that the share of scattering acts influencing energy transfer is n while that influencing the momentum transfer is m, we can calculate the mean real relaxation time from the relation

$$\ln\left(\frac{\bar{\tau}_r}{\tau_{max}}\right) = n \ln\left(\frac{\bar{\tau}_0}{\tau_{max}}\right) \tag{2}$$

in the case of energy transfer and from

$$\ln\left(\frac{\bar{\tau}_r}{\tau_{max}}\right) = m \ln\left(\frac{\bar{\tau}_0}{\tau_{max}}\right) \tag{3}$$

in the case of momentum transfer.

Thus, it is obvious that the values of n and m must lie within the limits of from zeto to unity.

Based on this model, we can determine the temperature-dependent thermal conductivity (λ) and dynamic viscosity (μ) for any substances in any aggregate form as

$$\lambda = \frac{1}{3} C\rho v_s^2 \bar{\tau}_r , \tag{4}$$

$$\mu = \frac{1}{3} \rho v_s^2 \bar{\tau}_r , \tag{5}$$

where $\bar{\tau}_r$ in (4) is determined from (2), while in formula (5) from (3), C is heat capacity and ρ is density.

Determination of the numerical values of n and m by these relations can serve as a check of the validity of the latter. Using numerous literature data and our experimental studies in the field of interest, we have calculated these quantities for a great number of chemical elements, semiconducting materials, high-alloy steels, metal oxides, coolants, water, and air in all three aggregate states. For all these substances in the temperature range of from 1 K to the melting temperature in solids, from the melting temperature to the boiling temperature in liquids, and in the gaseous state to temperatures considerably exceeding the boiling temperature, the values of n and m lie within the limits of from 0 to 1 [2-4]. Characteristic temperature dependences of these quantities are represented in Fig. 1.

As is seen, the character of scattering in momentum and energy transfer determined by the n and m values is fully consistent with the known qualitative representations of these processes. The methods of calculating n and m with respect to the structure of substances and their aggregate states need special investigation based on statistical physics, theory of fractals, and deterministic chaos.

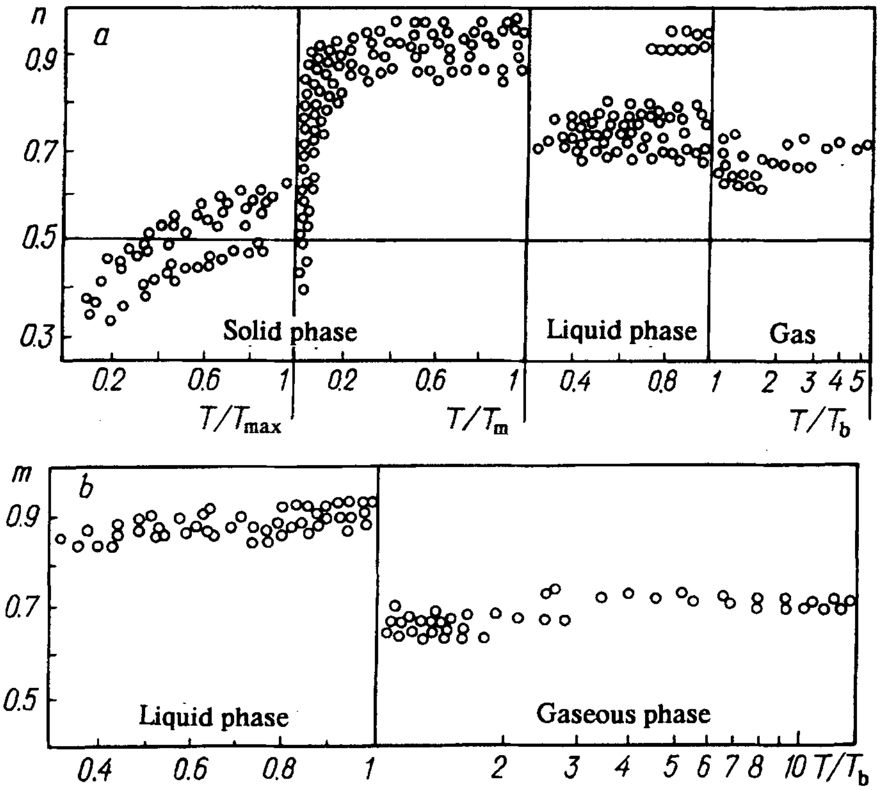

Fig. 1. Temperature dependence of parameters n (a) and m (b) of proposed relaxation model (T_{max}, T_m, T_b are the temperatures of maximum thermal conductivity, melting, and boiling of the substance, respectively).

It is pertinent to note, however, some interesting conclusions that can be drawn from an analysis of the relations obtained. It has been mentioned above that some scattering processes can influence momentum transfer without exerting an influence on energy transfer. This explains the observed excess of m over n at the same temperatures. Remembering that the Prandtl numbers are defined as $\mathrm{Pr} = \nu/a$, it is easy to obtain the following relation

$$\mathrm{Pr} = \left(\frac{\bar{\tau}_r}{\tau_{max}}\right)^{m-n} . \tag{6}$$

Calculations of the Prandtl numbers of different substances by this relation have shown good consistency with the literature data.

If we assume that n and m are equal to zero (i.e., no scattering processes occur in the substance), the thermal conductivity and viscosity at the given temperature are seen to reach their maximum possible values. But when a solid temperature is equal to the characteristic one, the maximum possible thermal conductivity of the solid is observed, which is determined only by its heat capacity, density, and sound velocity. Therefore, the proposed model allows determination of the maximum thermal conductivity of real substances. This problem has not been substantiated in the literature until now.

It should be noted that formally relations (4), (5), and (6) can be also used for determination and analysis of turbulent thermal conductivity, viscosity, and the turbulent Prandtl number. The calculations made show that in this case n_t and m_t lie within the limits of from 0 to 1 and at all temperatures are lower than the molecular coefficients. This is apparently explained by the fact that the turbulent coefficients contain velocity fluctuation components, which exert an influence on transfer and scattering processes. In this case, relations (4), (5), and (6) should be considered mainly empirical, since the turbulent transfer coefficients are functions of the flow and heat-transfer conditions rather than of the physical properties of the medium. Nevertheless, analysis of the mechanism of turbulent transfer on the basis of scattering processes by the proposed model seems to be promising, since it makes it possible, by employing classical representations of thermal conductivity, viscosity, and Prandtl number, to express peculiar features of their turbulent analogs in terms of a scattering mechanism on the basis of an analysis of changes in the frequency characteristics and wave vectors with allowance for velocity and temperature fluctuations imposed on them.

In conclusion, it is also worth noting that the model parameters n and m as shares of actual scattering processes determining energy and momentum transfer can also be found from experimental thermal conductivity and viscosity values which allows one to give a sufficiently adequate quantitative interpretation of these phenomena, which are difficult to calculate exactly by the quantitative methods of modern theoretical physics.

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